Quantum BRST charge and $OSp(1|8)$ superalgebra of twistor-like p-branes with exotic supersymmetry and Weyl symmetry

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Abstract

Algebra of the constraints of twistor-like p-branes restoring $\frac{3}{4}$ fraction of the spontaneously broken $D = 4$ $N = 1$ supersymmetry is studied using the conversion method. Classical and quantum realizations of the BRST charge, unified superalgebra of the global generalized superconformal $OSp(1|8)$ and Virasoro and Weyl symmetries are constructed. It is shown that the quantum Hermitian BRST charge is nilpotent and the quantized $OSp(1|8)$ superalgebra is closed.

1 Introduction

The physical interpretation of the central charges in supersymmetry algebra as topological charges carried by branes [1] advanced understanding of the phenomena of partial spontaneous breaking of supersymmetry [2]. Because branes are constituents of M-theory, spontaneously breaking supersymmetry, their global and local symmetries correlate with the symmetries of M-theory [3], [4]. Studying these symmetries resulted in the model independent classical analysis of BPS states preserving $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ fractions of the partially spontaneously broken $D = 4$ $N = 1$ supersymmetry [5]. A special interest to construction of a physical model with domain wall configurations preserving $\frac{3}{4}$ fraction of the $D = 4$ $N = 1$ supersymmetry against spontaneous breaking was subscribed there. That configurations were earlier studied in superparticle dynamics [6] and algebraically realized as the brane intersections in [7]. Then the tensionless string/brane model preserving $\frac{3}{4}$ fraction of the $D = 4$ $N = 1$ supersymmetry and generating static solutions for these tensionless objects was proposed in [8]. These results have sharpen the general question: whether quantum exotic BPS states saturated by the p-brane states protect the same high $\frac{M-1}{M}$ fraction of $N = 1$ global supersymmetry against spontaneous breaking as in the classical case?

We have started studying the question in [9] on the example of the p-branes preserving $\frac{3}{4}$ fraction of the partially spontaneously broken $D = 4$ $N = 1$ supersymmetry and found some obstacles for the quantization in the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering previously studied in [10] (see also [11]). Here we analyze the quantization problem applying the BFV approach [12] and construct quantum Hermitian BRST operator and the generators of gauge Weyl, Virasoro and global $OSp(1|8)$ symmetries extended by the ghost contributions. We prove the nilpotency of the

quantum Hermitian BRST charge, its (anti)commutativity with the quantum Hermitian generators of the $OSp(1|8)$ superalgebra and the closure of this quantized superlgebra. At the same time we show that the quantum $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered BRST operator and the $OSp(1|8)$ generators are nonHermitian and differ from the Hermitian ones by the presence of the divergent terms. We discuss a possibility to overcome this obstacle by the choice of the special regularization prescription for the p-brane world-volume delta function $\delta^p(\vec{\sigma}-\vec{\sigma}')|_{\vec{\sigma}=\vec{\sigma}'}$ and its derivative $\partial_M \delta^p(0)$. A possibility of the exact cancellation of the divergent terms in the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered quantum operators for other dimensions $D = 2, 3, 4 (mod 8)$ is also discussed.

2 Conversion of tensionless super p-brane constraints

New models of tensionless string and p-branes evolving in the symplectic superspace $\mathcal{M}_{\mathsf{M}}^{susy}$ and preserving all but one fractions of $N = 1$ supersymmetry were recently studied in [8], [13]. For $\mathsf{M} = 2^{[\frac{D}{2}]}$ ($D = 2, 3, 4 \ mod \ 8$) the space $\mathcal{M}_{\mathsf{M}}^{susy}$ extends the standard D-dimensional super space-time (x_{ab}, θ_a), (where $a, b = 1, 2, ..., 2^{[\frac{D}{2}]}$) by the tensor central charge (TCC) coordinates z_{ab}. The coordinates $x_{ab} = x^m(\gamma_m C^{-1})_{ab}$ and $z_{ab} = iz^{mn}(\gamma_{mn}C^{-1})_{ab} + z^{mnl}(\gamma_{mnl}C^{-1})_{ab} + ...$ constitute components of the symmetric spin-tensor Y_{ab}. In terms of Y_{ab} and the Majorana spinor θ_a the action [8], invariant under the spontaneously broken $N = 1$ supersymmetry and world-volume reparametrizations, is given by

$$S_p = \frac{1}{2} \int d\tau d^p\sigma \ \rho^\mu U^a W_{\mu ab} U^b, \tag{1}$$

where $W_{ab} = W_{\mu ab} d\xi^\mu$ is the supersymmetric Cartan differential one-form

$$W_{\mu ab} = \partial_\mu Y_{ab} - 2i(\partial_\mu \theta_a \theta_b + \partial_\mu \theta_b \theta_a), \tag{2}$$

and $\partial_\mu \equiv \frac{\partial}{\partial \xi^\mu}$ with $\xi^\mu = (\tau, \sigma^M)$, ($M = 1, 2, ..., p$) parametrizing the p-brane world volume. The local auxiliary Majorana spinor $U^a(\tau, \sigma^M)$ parametrizes the generalized momentum $P^{ab} = \frac{1}{2}\rho^\tau U^a U^b$ of tensionless p-brane and $\rho^\mu(\tau, \sigma^M)$ is the world-volume vector density providing the reparametrization invariance of S_p similarly to the null branes [14].

The action (1) has $(\mathsf{M} - 1)$ κ−symmetries

$$\delta_\kappa \theta_a = \kappa_a, \quad \delta_\kappa Y_{ab} = -2i(\theta_a \kappa_b + \theta_b \kappa_a), \quad \delta_\kappa U^a = 0, \tag{3}$$

which protect $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of the $N = 1$ global supersymmetry to be spontaneously broken, because of the one real condition $U^a \kappa_a = 0$ for the transformation parameters $\kappa_a(\tau, \vec{\sigma})^1$.

For the four-dimensional space-time the action (1) takes the form

$$S_p = \frac{1}{2} \int d\tau d^p\sigma \ \rho^\mu \left(2u^\alpha \omega_{\mu\alpha\dot\alpha} \bar{u}^{\dot\alpha} + u^\alpha \omega_{\mu\alpha\beta} u^\beta + \bar{u}^{\dot\alpha} \bar{\omega}_{\mu\dot\alpha\dot\beta} \bar{u}^{\dot\beta} \right). \tag{4}$$

This action is invariant under the $OSp(1|8)$ symmetry, which is global supersymmetry of the massless fields of all spins in $D = 4$ space-time extended by TCC coordinates [15], [16].

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[1]To remove a possible misunderstanding in this terminology let us remind that from the world-volume perspective the last fraction of the $N = 1$ supersymmetry is also the symmetry of the action (1), but it is spontaneously broken, because of the presence of Goldstone fermion $\tilde{\eta} = -2i(U^a \theta_a)$ encoding the single physical fermionic degree of freedom associated with θ_a (see [13] for details).

The Hamiltonian structure of the action (4), described in [17], is characterized by 3 fermionic and $2p+7$ bosonic first-class constraints that generate its local symmetries, as well as, 1 fermionic and 8 bosonic second-class constraints taken into account by the construction of the Dirac bracket. We found that the D.B. algebra of the first-class constraints has the rank equal two and it gives rise to the higher powers of the ghosts in the BRST generator.

To simplify transition to the quantum theory the conversion method [18]-[23], transforming all the primary and secondary constraints to the first class, has been applied in [9]. To this end the additional canonically conjugate pairs (P_q^α, q^α), $(\bar{P}_q^{\dot\alpha}, \bar{q}^{\dot\alpha})$, $(P_\tau^{(\varphi)}, \varphi^\tau)$ and the self-conjugate Grassmannian variable f have been introduced. As a result, all the constraints have been converted to the effective first-class constraints in the extended phase space.

The converted constraints for the auxiliary fields are the following

$$\widetilde{P}_u^\alpha = P_u^\alpha + P_q^\alpha \approx 0, \quad \widetilde{\bar{P}}_u^{\dot\alpha} = \bar{P}_u^{\dot\alpha} + \bar{P}_q^{\dot\alpha} \approx 0, \tag{5}$$

$$\widetilde{P}_\tau^{(\rho)} = P_\tau^{(\rho)} + P_\tau^{(\varphi)} \approx 0, \quad P_M^{(\rho)} \approx 0. \tag{6}$$

The converted bosonic constraints $\widetilde{\Phi} \equiv (\widetilde{\Phi}^{\dot\alpha\alpha}, \widetilde{\Phi}^{\alpha\beta}, \widetilde{\bar{\Phi}}^{\dot\alpha\dot\beta})$ originating from the Φ-constraints [9] are given by

$$\widetilde{\Phi}^{\dot\alpha\alpha} = P^{\dot\alpha\alpha} - \tilde{\rho}^\tau \tilde{u}^\alpha \bar{\tilde{u}}^{\dot\alpha} \approx 0, \ \widetilde{\Phi}^{\alpha\beta} = \pi^{\alpha\beta} + \frac{1}{2}\tilde{\rho}^\tau \tilde{u}^\alpha \tilde{u}^\beta \approx 0, \ \widetilde{\bar{\Phi}}^{\dot\alpha\dot\beta} = \bar{\pi}^{\dot\alpha\dot\beta} + \frac{1}{2}\tilde{\rho}^\tau \bar{\tilde{u}}^{\dot\alpha} \bar{\tilde{u}}^{\dot\beta} \approx 0 \tag{7}$$

and have zero Poisson brackets (P.B.) with the constraints (5), (6) and among themselves. The converted fermionic constraints $\widetilde{\Psi} = (\widetilde{\Psi}^\alpha, \widetilde{\bar{\Psi}}^{\dot\alpha})$ originating from the primary Ψ-constraints and generating four $\kappa-$symmetries take the form

$$\widetilde{\Psi}^\alpha = \pi^\alpha - 2i\bar{\theta}_{\dot\alpha}P^{\dot\alpha\alpha} - 4i\pi^{\alpha\beta}\theta_\beta + 2(\tilde{\rho}^\tau)^{1/2}\tilde{u}^\alpha f \approx 0,$$
$$\widetilde{\bar{\Psi}}^{\dot\alpha} = -(\widetilde{\Psi}^\alpha)^* = \bar{\pi}^{\dot\alpha} - 2iP^{\dot\alpha\alpha}\theta_\alpha - 4i\bar{\pi}^{\dot\alpha\dot\beta}\bar{\theta}_{\dot\beta} - 2(\tilde{\rho}^\tau)^{1/2}\bar{\tilde{u}}^{\dot\alpha}f \approx 0, \tag{8}$$

where $f^* = f$ is an auxiliary Grassmannian variable characterized by the P.B.

$$\{f(\vec{\sigma}), f(\vec{\sigma}')\}_{P.B.} = -i\delta^p(\vec{\sigma} - \vec{\sigma}'). \tag{9}$$

The addition of the field $f(\tau, \vec{\sigma})$ restores the forth κ-symmetry and transforms all $\widetilde{\Psi}$-constraints to the first class. The Weyl symmetry constraint $\widetilde{\Delta}_W$ in the extended phase space is

$$\widetilde{\Delta}_W = (\widetilde{\bar{P}}_u^\alpha \tilde{u}_\alpha + \widetilde{\bar{P}}_u^{\dot\alpha} \bar{\tilde{u}}_{\dot\alpha}) - 2\tilde{\rho}^\tau \widetilde{P}_\tau^{(\rho)} - 2\rho^M P_M^{(\rho)} \approx 0, \tag{10}$$

where the variables $(\tilde{u}^\alpha = u^\alpha - q^\alpha, \ \widetilde{\bar{P}}_u^\alpha = \frac{1}{2}(P_u^\alpha - P_q^\alpha))$ and $(\tilde{\rho}^\tau = \rho^\tau - \varphi^\tau, \ \widetilde{P}_\tau^{(\rho)} = \frac{1}{2}(P_\tau^{(\rho)} - P_\tau^{(\varphi)}))$ form canonically conjugate pairs [9]. Finally, the converted constraints $\widetilde{L}_M$ of the world-volume $\vec{\sigma}-$reparametrizations are

$$\begin{aligned}\widetilde{L}_M = & \ \partial_M x_{\alpha\dot\alpha} P^{\dot\alpha\alpha} + \partial_M z_{\alpha\beta}\pi^{\alpha\beta} + \partial_M \bar{z}_{\dot\alpha\dot\beta}\bar{\pi}^{\dot\alpha\dot\beta} + \partial_M \theta_\alpha \pi^\alpha + \partial_M \bar{\theta}_{\dot\alpha}\bar{\pi}^{\dot\alpha} \\ + & \ \partial_M \tilde{u}_\alpha \widetilde{\bar{P}}_u^\alpha + \partial_M \bar{\tilde{u}}_{\dot\alpha}\widetilde{\bar{P}}_u^{\dot\alpha} - \tilde{\rho}^\tau \partial_M \widetilde{P}_\tau^{(\rho)} - \rho^N \partial_M P_N^{(\rho)} - \frac{i}{2}f\partial_M f \approx 0. \end{aligned} \tag{11}$$

The P.B. superalgebra of the converted first-class constraints (5)-(8), (10), (11) is described by the following non zero relations

$$\{\widetilde{\Psi}^\alpha(\vec{\sigma}), \widetilde{\Psi}^\beta(\vec{\sigma}')\}_{P.B.} = -8i\widetilde{\Phi}^{\alpha\beta}\delta^p(\vec{\sigma} - \vec{\sigma}'), \tag{12}$$

3

$$\{\widetilde{\Psi}^{\alpha}(\vec{\sigma}), \bar{\widetilde{\Psi}}^{\dot{\beta}}(\vec{\sigma}')\}_{P.B.} = -4i\widetilde{\Phi}^{\dot{\beta}\alpha}\delta^p(\vec{\sigma} - \vec{\sigma}'), \tag{13}$$

$$\{\widetilde{\Delta}_W(\vec{\sigma}), P_M^{(\rho)}(\vec{\sigma}')\}_{P.B.} = 2P_M^{(\rho)}\delta^p(\vec{\sigma} - \vec{\sigma}'), \tag{14}$$

$$\{\widetilde{L}_M(\vec{\sigma}), P_N^{(\rho)}(\vec{\sigma}')\}_{P.B.} = \partial_M P_N^{(\rho)}\delta^p(\vec{\sigma} - \vec{\sigma}'), \tag{15}$$

$$\{\widetilde{L}_M(\vec{\sigma}), \widetilde{L}_N(\vec{\sigma}')\}_{P.B.} = (\widetilde{L}_M(\vec{\sigma}')\partial_{N'} - \widetilde{L}_N(\vec{\sigma})\partial_M)\delta^p(\vec{\sigma} - \vec{\sigma}'), \tag{16}$$

$$\{\widetilde{L}_M(\vec{\sigma}), \chi(\vec{\sigma}')\}_{P.B.} = -\chi(\vec{\sigma})\partial_M\delta^p(\vec{\sigma} - \vec{\sigma}'), \tag{17}$$

where χ are $\widetilde{\Phi}$, $\widetilde{\Psi}$ and $\widetilde{\Delta}_W$ constraints. The complex conjugate relations have to be added to (12)-(17). The remaining P.B.'s of the constraints are equal to zero in the strong sense. Having the algebra (12)-(17) one can construct BRST charge of the tensionless super p-brane.

3   BRST charge and $OSp(1|8)$ symmetry generators

The algebra(12)-(17) has the rank equal unity and may be presented in the generalized canonical form

$$\{\Upsilon^{\mathcal{A}}(\vec{\sigma}), \Upsilon^{\mathcal{B}}(\vec{\sigma}')\}_{P.B.} = \int d^p\sigma'' f^{\mathcal{AB}}{}_{\mathcal{C}}(\vec{\sigma}, \vec{\sigma}'|\vec{\sigma}'')\Upsilon^{\mathcal{C}}(\vec{\sigma}''), \tag{18}$$

where $f^{\mathcal{AB}}{}_{\mathcal{C}}$ are structure functions. Let us note that the algebra (18) generalizes the original algebra [12] by the taking into account $\partial_M\delta^p(\vec{\sigma} - \vec{\sigma}')$ in the structure functions following from the P.B.'s including the Virasoro constraints $\widetilde{L}_M(\vec{\sigma})$ such as

$$f^{\widetilde{L}_M\widetilde{\Phi}^{\dot{\alpha}\beta}}{}_{\widetilde{\Phi}^{\dot{\gamma}\delta}} = -\delta^{\dot{\alpha}}_{\dot{\gamma}}\delta^{\beta}_{\delta}\partial_M\delta^p(\vec{\sigma} - \vec{\sigma}')\delta^p(\vec{\sigma} - \vec{\sigma}''),$$

$$f^{\widetilde{L}_M\widetilde{L}_N}{}_{\widetilde{L}_Q} = -\delta^Q_N\partial_M\delta^p(\vec{\sigma} - \vec{\sigma}')\delta^p(\vec{\sigma} - \vec{\sigma}'') + \delta^Q_M\partial_{N'}\delta^p(\vec{\sigma} - \vec{\sigma}')\delta^p(\vec{\sigma}' - \vec{\sigma}'') \tag{19}$$

and other ones.

The canonically conjugate ghost pairs of the minimal sector corresponding to the first-class constraints may be introduced forming the following triads

$$(\widetilde{\Phi}^{\alpha\beta}, C_{\alpha\beta}, \check{P}^{\alpha\beta}); \quad (\bar{\widetilde{\Phi}}^{\dot{\alpha}\dot{\beta}}, \bar{C}_{\dot{\alpha}\dot{\beta}}, \bar{\check{P}}^{\dot{\alpha}\dot{\beta}}); \quad (\widetilde{\Phi}^{\dot{\alpha}\beta}, C_{\beta\dot{\alpha}}, \check{P}^{\dot{\alpha}\beta});$$

$$(\widetilde{\Psi}^{\alpha}, C_{\alpha}, \check{P}^{\alpha}); \quad (\bar{\widetilde{\Psi}}^{\dot{\alpha}}, \bar{C}_{\dot{\alpha}}, \bar{\check{P}}^{\dot{\alpha}});$$

$$(\tilde{P}_u^{\alpha}, C_{u\alpha}, \check{P}_u^{\alpha}); \quad (\bar{\tilde{P}}_u^{\dot{\alpha}}, \bar{C}_{u\dot{\alpha}}, \bar{\check{P}}_u^{\dot{\alpha}});$$

$$(\tilde{P}_\tau^{(\rho)}, C^{(\rho)\tau}, \check{P}_\tau^{(\rho)}); \quad (P_M^{(\rho)}, C^{(\rho)M}, \check{P}_M^{(\rho)});$$

$$(\widetilde{L}_M, C^M, \check{P}_M); \quad (\widetilde{\Delta}_W, C^{(W)}, \check{P}^{(W)}). \tag{20}$$

Utilizing nonzero structure functions of the superalgebra (12)-(17) one can present the corresponding BRST generator Ω of the minimal sector [12]

$$\Omega = \int d^p\sigma (C_{\mathcal{A}}\Upsilon^{\mathcal{A}} + \tfrac{1}{2}(-)^b C_{\mathcal{B}} C_{\mathcal{A}} \check{f}^{\mathcal{AB}}{}_{\mathcal{C}} \check{P}^{\mathcal{C}})(\vec{\sigma}), \tag{21}$$

by the following integral along the hypersurface of the closed super p-brane

$$\Omega = \int d^p\sigma \left[(C_{\alpha\beta}\widetilde{\Phi}^{\alpha\beta} + \bar{C}_{\dot{\alpha}\dot{\beta}}\bar{\widetilde{\Phi}}^{\dot{\alpha}\dot{\beta}} + C_\alpha\widetilde{\Psi}^\alpha - \bar{C}_{\dot{\alpha}}\bar{\widetilde{\Psi}}^{\dot{\alpha}} + C_{u\alpha}\tilde{P}_u^\alpha + \bar{C}_{u\dot{\alpha}}\bar{\tilde{P}}_u^{\dot{\alpha}}) \right.$$

$$+ C_{\alpha\dot{\beta}}\widetilde{\Phi}^{\dot{\beta}\alpha} + C^{(\rho)\tau}\tilde{P}_\tau^{(\rho)} + C^{(\rho)M}P_M^{(\rho)} + C^{(W)}\widetilde{\Delta}_W^{ext} + C^M\widetilde{L}_M^{ext}$$

$$\left. + 4i(C_\alpha C_\beta \check{P}^{\alpha\beta} + \bar{C}_{\dot{\alpha}}\bar{C}_{\dot{\beta}}\bar{\check{P}}^{\dot{\alpha}\dot{\beta}}) - 4iC_\alpha\bar{C}_{\dot{\beta}}\check{P}^{\dot{\beta}\alpha} \right]. \tag{22}$$

$\widetilde{\Delta}_W^{ext}$ in eq.(22) is the generator of the gauge world-volume Weyl symmetry

$$\widetilde{\Delta}_W^{ext} = \widetilde{\Delta}_W - 2C^{(\rho)M}\breve{P}_M^{(\rho)}, \qquad (23)$$

and $\widetilde{L}_M^{ext}$ is the generalized Virasoro generator

$$\begin{aligned}
\widetilde{L}_M^{ext} =\ & \widetilde{L}_M + \partial_M C_{\alpha\beta}\breve{P}^{\alpha\beta} + \partial_M \bar{C}_{\dot{\alpha}\dot{\beta}}\bar{\breve{P}}^{\dot{\alpha}\dot{\beta}} + \partial_M C_{\alpha\dot{\beta}}\breve{P}^{\dot{\beta}\alpha} \\
+\ & \partial_M C_\alpha \breve{P}^\alpha + \partial_M \bar{C}_{\dot{\alpha}}\bar{\breve{P}}^{\dot{\alpha}} + \partial_M C^{(W)}\breve{P}^{(W)} - C^{(\rho)N}\partial_M \breve{P}_N^{(\rho)} + \partial_M C^N \breve{P}_N
\end{aligned} \qquad (24)$$

extended by the ghost contributions.

Using the P.B.'s of the superalgebra (12)-(17) one can show that the P.B. of the BRST generator density $\Omega(\tau, \vec{\sigma})$, defined by the integrand (22), with itself is equal to the total derivative

$$\begin{aligned}
\{\Omega(\vec{\sigma}), \Omega(\vec{\sigma}')\}_{P.B.} =\ & \partial_M(C^M(C_{\alpha\beta}\widetilde{\Phi}^{\alpha\beta} + \bar{C}_{\dot{\alpha}\dot{\beta}}\bar{\widetilde{\Phi}}^{\dot{\alpha}\dot{\beta}} + C_{\alpha\dot{\beta}}\widetilde{\Phi}^{\dot{\beta}\alpha} \\
+\ & C_\alpha \widetilde{\Psi}^\alpha - \bar{C}_{\dot{\alpha}}\bar{\widetilde{\Psi}}^{\dot{\alpha}} + C^{(\rho)N}P_N^{(\rho)} + C^{(W)}\widetilde{\Delta}_W^{ext} + C^N \widetilde{L}_N^{ext} \\
+\ & 4i(C_\alpha C_\beta \breve{P}^{\alpha\beta} + \bar{C}_{\dot{\alpha}}\bar{C}_{\dot{\beta}}\breve{P}^{\dot{\alpha}\dot{\beta}} - C_\alpha \bar{C}_{\dot{\beta}}\breve{P}^{\dot{\beta}\alpha}))\delta^p(\vec{\sigma} - \vec{\sigma}'),
\end{aligned} \qquad (25)$$

because of the presence of $\partial_M \delta^p(\vec{\sigma} - \vec{\sigma}')$ in the structure functions of the superalgebra (12)-(17). But, the contribution of the total derivative in the r.h.s. of (25) vanishes after integration in $\vec{\sigma}$ and $\vec{\sigma}'$ due to the periodical boundary conditions for the closed p-brane. It results in the P.B.-anticommutativity of the BRST charge $\Omega \equiv \int d^p\sigma \Omega(\tau, \vec{\sigma})$ (21) with itself

$$\{\Omega, \Omega\}_{P.B.} = 0. \qquad (26)$$

The introduction of the ghost variables leads to the extension of the $OSp(1|8)$ symmetry generators providing the P.B.-(anti)commutativity of the $OSp(1|8)$ generators with Ω (21). The ghost extended "square roots" $\widetilde{S}_\gamma(\tau, \vec{\sigma})$ and $\bar{\widetilde{S}}_{\dot{\gamma}}(\tau, \vec{\sigma})$ of the ghost extended conformal boost densities $\widetilde{K}_{\gamma\dot{\gamma}}(\tau, \vec{\sigma})$ and $\widetilde{K}_{\gamma\lambda}(\tau, \vec{\sigma})$ are given by

$$\begin{aligned}
\widetilde{S}_\gamma(\tau, \vec{\sigma}) =\ & (z_{\gamma\delta} - 2i\theta_\gamma\theta_\delta)Q^\delta + (x_{\gamma\dot{\delta}} - 2i\theta_\gamma\bar{\theta}_{\dot{\delta}})\bar{Q}^{\dot{\delta}} + 4i(\tilde{u}^\delta\theta_\delta - \bar{\tilde{u}}^{\dot{\delta}}\bar{\theta}_{\dot{\delta}})\tilde{\bar{P}}_{u\gamma} + \frac{2}{(\tilde{\rho}^\tau)^{1/2}}\tilde{\bar{P}}_{u\gamma}f \\
& + C_\gamma{}^\beta \breve{P}_\beta + C_{\gamma\dot{\beta}}\bar{\breve{P}}^{\dot{\beta}} + 4iC_\gamma(\theta_\beta\breve{P}^\beta - \bar{\theta}_{\dot{\beta}}\bar{\breve{P}}^{\dot{\beta}}) \\
& - 8i\theta_\delta C_\gamma{}^\beta \breve{P}_\beta{}^\delta + 4i\bar{\theta}^{\dot{\delta}}C_\gamma{}^\beta \breve{P}_{\beta\dot{\delta}} + 4i\theta_\delta C_{\gamma\dot{\beta}}\breve{P}^{\dot{\beta}\delta} - 8i\bar{\theta}^{\dot{\delta}}C_{\gamma\dot{\beta}}\bar{\breve{P}}^{\dot{\beta}}{}_{\dot{\delta}},
\end{aligned} \qquad (27)$$

$$\begin{aligned}
\bar{\widetilde{S}}_{\dot{\gamma}}(\tau, \vec{\sigma}) =\ & (\bar{z}_{\dot{\gamma}\dot{\delta}} - 2i\bar{\theta}_{\dot{\gamma}}\bar{\theta}_{\dot{\delta}})\bar{Q}^{\dot{\delta}} + (x_{\delta\dot{\gamma}} + 2i\theta_\delta\bar{\theta}_{\dot{\gamma}})Q^\delta - 4i(\tilde{u}^\delta\theta_\delta - \bar{\tilde{u}}^{\dot{\delta}}\bar{\theta}_{\dot{\delta}})\bar{\tilde{P}}_{u\dot{\gamma}} - \frac{2}{(\tilde{\rho}^\tau)^{1/2}}\bar{\tilde{P}}_{u\dot{\gamma}}f \\
& - C_{\beta\dot{\gamma}}\breve{P}^\beta - \bar{C}^{\dot{\beta}}{}_{\dot{\gamma}}\bar{\breve{P}}_{\dot{\beta}} - 4i\bar{C}_{\dot{\gamma}}(\theta_\beta\breve{P}^\beta - \bar{\theta}_{\dot{\beta}}\bar{\breve{P}}^{\dot{\beta}}) \\
& - 8i\theta^\delta C_{\beta\dot{\gamma}}\breve{P}_\delta{}^\beta + 4i\bar{\theta}_{\dot{\delta}}C_{\beta\dot{\gamma}}\breve{P}^{\dot{\delta}\beta} + 4i\theta^\delta \bar{C}^{\dot{\beta}}{}_{\dot{\gamma}}\breve{P}_{\delta\dot{\beta}} - 8i\bar{\theta}_{\dot{\delta}}\bar{C}^{\dot{\beta}}{}_{\dot{\gamma}}\bar{\breve{P}}^{\dot{\delta}}{}_{\dot{\beta}}.
\end{aligned} \qquad (28)$$

Using the densities $\Omega(\tau, \vec{\sigma})$ (22) and $\widetilde{S}_\gamma(\tau, \vec{\sigma}')$ (27) we find their P.B.

$$\{\Omega(\sigma), \widetilde{S}_\gamma(\vec{\sigma}')\}_{P.B.} = -(C^M \widetilde{S}_\gamma)(\vec{\sigma})\partial_M \delta^p(\sigma - \sigma') \qquad (29)$$

and conclude that the contribution of the total derivative in the r.h.s. of (29) vanishes after integration with respect to $\vec{\sigma}$ and $\vec{\sigma}'$. Thus, the BRST charge Ω (22) has zero P.B.'s with the conformal supercharges $\widetilde{S}_\gamma, \bar{\widetilde{S}}_{\dot{\gamma}}$

$$\{\Omega, \widetilde{S}_\gamma\}_{P.B.} = 0, \quad \{\Omega, \bar{\widetilde{S}}_{\dot{\gamma}}\}_{P.B.} = 0. \qquad (30)$$

The same P.B.-commutativity

$$\{\Omega, G\}_{P.B.} = 0 \tag{31}$$

between Ω and other $OSp(1|8)$ symmetry charges $G \equiv \int d^p\sigma G(\tau, \vec{\sigma})$ extended by the ghost contributions will also be preserved, because of the general relation (17) for the generator densities: $\{\widetilde{L}_M(\vec{\sigma}), G(\vec{\sigma}')\}_{P.B.} = -G(\vec{\sigma})\partial_M\delta(\vec{\sigma} - \vec{\sigma}')$.

The above mentioned expressions for the generator densities of the generalized conformal transformations extended by the ghost contributions take the form[2]

$$
\begin{aligned}
\widetilde{K}_{\gamma\lambda}(\tau, \vec{\sigma}) =\ & 2z_{\gamma\beta}z_{\lambda\delta}\pi^{\beta\delta} + 2x_{\gamma\dot\beta}x_{\lambda\dot\delta}\bar\pi^{\dot\beta\dot\delta} + z_{\gamma\beta}x_{\lambda\dot\delta}P^{\dot\delta\beta} + x_{\gamma\dot\beta}z_{\lambda\delta}P^{\dot\beta\delta} \\
& + \theta_\lambda(z_{\gamma\delta}\pi^\delta + x_{\gamma\dot\delta}\bar\pi^{\dot\delta}) + \theta_\gamma(z_{\lambda\delta}\pi^\delta + x_{\lambda\dot\beta}\bar\pi^{\dot\delta}) + (\tilde u^\delta z_{\delta\lambda} - \bar{\tilde u}^{\dot\delta}x_{\lambda\dot\delta})\tilde{\breve P}_{u\gamma} + (\tilde u^\delta z_{\delta\gamma} - \bar{\tilde u}^{\dot\delta}x_{\gamma\dot\delta})\tilde{\breve P}_{u\lambda} \\
& - 2i(\tilde u^\delta\theta_\delta - \bar{\tilde u}^{\dot\delta}\bar\theta_{\dot\delta})(\theta_\lambda\tilde{\breve P}_{u\gamma} + \theta_\gamma\tilde{\breve P}_{u\lambda}) + \tfrac{2}{(\tilde\rho^\tau)^{1/2}}(\theta_\lambda\tilde{\breve P}_{u\gamma} + \theta_\gamma\tilde{\breve P}_{u\lambda})f - \tfrac{1}{\tilde\rho^\tau}\tilde{\breve P}_{u\gamma}\tilde{\breve P}_{u\lambda} \\
& + \theta_\lambda C_\gamma{}^\beta\breve P_\beta + \theta_\gamma C_\lambda{}^\beta\breve P_\beta + \theta_\lambda C_{\gamma\dot\beta}\bar{\breve P}^{\dot\beta} + \theta_\gamma C_{\lambda\dot\beta}\bar{\breve P}^{\dot\beta} \\
& - (z_\lambda{}^\beta + 2i\theta_\lambda\theta^\beta)C_\gamma\breve P_\beta - (z_\gamma{}^\beta + 2i\theta_\gamma\theta^\beta)C_\lambda\breve P_\beta - (x_{\lambda\dot\beta} + 2i\theta_\lambda\bar\theta_{\dot\beta})C_\gamma\bar{\breve P}^{\dot\beta} - (x_{\gamma\dot\beta} + 2i\theta_\gamma\bar\theta_{\dot\beta})C_\lambda\bar{\breve P}^{\dot\beta} \\
& - 2(z_\gamma{}^\beta + 2i\theta_\gamma\theta^\beta)C_{\lambda\delta}\breve P_\beta{}^\delta - 2(z_\lambda{}^\beta + 2i\theta_\lambda\theta^\beta)C_{\gamma\delta}\breve P_\beta{}^\delta \\
& + (z_{\gamma\beta} + 2i\theta_\gamma\theta_\beta)C_{\lambda\dot\delta}\breve P^{\dot\delta\beta} + (z_{\lambda\beta} + 2i\theta_\lambda\theta_\beta)C_{\gamma\dot\delta}\breve P^{\dot\delta\beta} \\
& + (x_{\gamma\dot\beta} + 2i\theta_\gamma\bar\theta_{\dot\beta})C_{\lambda\delta}\breve P^{\dot\beta\delta} + (x_{\lambda\dot\beta} + 2i\theta_\lambda\bar\theta_{\dot\beta})C_{\gamma\delta}\breve P^{\dot\beta\delta} \\
& + 2(x_{\gamma\dot\beta} + 2i\theta_\gamma\bar\theta_{\dot\beta})C_{\lambda\dot\delta}\bar{\breve P}^{\dot\beta\dot\delta} + (x_{\lambda\dot\beta} + 2i\theta_\lambda\bar\theta_{\dot\beta})C_{\gamma\dot\delta}\bar{\breve P}^{\dot\beta\dot\delta},
\end{aligned} \tag{32}
$$

for $\widetilde{K}_{\gamma\lambda}(\tau, \vec{\sigma})$ and respectively for $\widetilde{K}_{\gamma\dot\gamma}(\tau, \vec{\sigma})$

$$
\begin{aligned}
\widetilde{K}_{\gamma\dot\gamma}(\tau, \vec{\sigma}) =\ & z_{\gamma\delta}\bar z_{\dot\gamma\dot\delta}P^{\dot\delta\delta} + x_{\gamma\dot\delta}x_{\delta\dot\gamma}P^{\dot\delta\delta} + 2(z_{\gamma\delta}x_{\lambda\dot\gamma}\pi^{\delta\lambda} + x_{\gamma\dot\delta}\bar z_{\dot\gamma\dot\lambda}\bar\pi^{\dot\delta\dot\lambda}) \\
& + \theta_\gamma(\bar z_{\dot\gamma\dot\delta}\bar\pi^{\dot\delta} + x_{\delta\dot\gamma}\pi^\delta) + \bar\theta_{\dot\gamma}(z_{\gamma\delta}\pi^\delta + x_{\gamma\dot\delta}\bar\pi^{\dot\delta}) + (\tilde u^\delta x_{\delta\dot\gamma} - \bar{\tilde u}^{\dot\delta}\bar z_{\dot\gamma\dot\delta})\tilde{\breve P}_{u\gamma} + (x_{\gamma\dot\delta}\bar{\tilde u}^{\dot\delta} - z_{\gamma\delta}\tilde u^\delta)\bar{\tilde{\breve P}}_{u\dot\gamma} \\
& - 2i(\tilde u^\delta\theta_\delta - \bar{\tilde u}^{\dot\delta}\bar\theta_{\dot\delta})(\bar\theta_{\dot\gamma}\tilde{\breve P}_{u\gamma} - \theta_\gamma\bar{\tilde{\breve P}}_{u\dot\gamma}) + \tfrac{2}{(\tilde\rho^\tau)^{1/2}}(\bar\theta_{\dot\gamma}\tilde{\breve P}_{u\gamma} - \theta_\gamma\bar{\tilde{\breve P}}_{u\dot\gamma})f + \tfrac{1}{\tilde\rho^\tau}\tilde{\breve P}_{u\gamma}\bar{\tilde{\breve P}}_{u\dot\gamma} \\
& + \bar\theta_{\dot\gamma}C_\gamma{}^\beta\breve P_\beta - \theta_\gamma\bar C_{\dot\gamma}{}^{\dot\beta}\bar{\breve P}_{\dot\beta} + \bar\theta_{\dot\gamma}C_{\gamma\dot\beta}\bar{\breve P}^{\dot\beta} - \theta_\gamma C_{\beta\dot\gamma}\breve P^\beta \\
& + (\bar z_{\dot\gamma}{}^{\dot\beta} + 2i\bar\theta_{\dot\gamma}\bar\theta^{\dot\beta})C_\gamma\bar{\breve P}_{\dot\beta} + (z_\gamma{}^\beta + 2i\theta_\gamma\theta^\beta)\bar C_{\dot\gamma}\breve P_\beta + (x_{\beta\dot\gamma} - 2i\theta_\beta\bar\theta_{\dot\gamma})C_\gamma\breve P^\beta + (x_{\gamma\dot\beta} + 2i\theta_\gamma\bar\theta_{\dot\beta})\bar C_{\dot\gamma}\bar{\breve P}^{\dot\beta} \\
& - 2(z_\gamma{}^\beta + 2i\theta_\gamma\theta^\beta)C_{\delta\dot\gamma}\breve P_\beta{}^\delta - 2(\bar z_{\dot\gamma}{}^{\dot\beta} + 2i\bar\theta_{\dot\gamma}\bar\theta^{\dot\beta})\bar C_{\gamma\dot\delta}\bar{\breve P}^{\dot\delta}{}_{\dot\beta} \\
& + (z_\gamma{}^\beta + 2i\theta_\gamma\theta^\beta)\bar C^{\dot\delta}{}_{\dot\gamma}\breve P_{\beta\dot\delta} + (\bar z^\beta{}_{\dot\gamma} + 2i\bar\theta_{\dot\gamma}\theta^\beta)C_\gamma{}^\delta\bar{\breve P}_{\delta\dot\beta} \\
& + (x_{\gamma\dot\beta} + 2i\theta_\gamma\bar\theta_{\dot\beta})C_{\delta\dot\gamma}\breve P^{\dot\beta\delta} + (x_{\beta\dot\gamma} - 2i\theta_\beta\bar\theta_{\dot\gamma})C_{\gamma\dot\delta}\breve P^{\dot\delta\beta} \\
& - 2(x_{\gamma\dot\beta} + 2i\theta_\gamma\bar\theta_{\dot\beta})\bar C^{\dot\delta}{}_{\dot\gamma}\bar{\breve P}^{\dot\beta}{}_{\dot\delta} - 2(x_{\beta\dot\gamma} - 2i\theta_\beta\bar\theta_{\dot\gamma})C_\gamma{}^\delta\breve P_\delta{}^\beta.
\end{aligned} \tag{33}
$$

---

[2]The discussed formulae would look more compact in the Majorana spinor representation. However, it makes more obscure the contribution of the TCC coordinates $z_{\alpha\beta}$ which presence is crucial for the exotic supersymmetry protection and generation of the new bosonic gauge symmetries (see [13]) generalizing the well known symmetries of the Penrose twistor approach originally formulated in $D = 4$ [24]. In the Majorana representation $z_{\alpha\beta}$ are encoded in the symmetric 4×4 matrix $Y_a{}^b \equiv Y_{ad}C^{db} = \begin{pmatrix} z_\alpha{}^\beta & x_{\alpha\dot\beta} \\ \tilde x^{\dot\alpha\beta} & \tilde z^{\dot\alpha}{}_{\dot\beta} \end{pmatrix}$ together with the space-time coordinates $x_{\alpha\dot\alpha}$ (see [8], [17]).

The remaining 16 generator densities of $OSp(1|8)$ supergroup extended by the ghosts are the following

$$
\begin{aligned}
\widetilde{L}^\alpha{}_\beta(\tau,\vec{\sigma}) &= x_{\beta\dot\beta}P^{\dot\beta\alpha} + 2z_{\gamma\beta}\pi^{\alpha\gamma} + \theta_\beta\pi^\alpha + \tilde{u}^\alpha\tilde{\breve{P}}_{u\beta} - 2C_\beta{}^\gamma\breve{P}_\gamma{}^\alpha + C_{\beta\dot\gamma}\breve{P}^{\dot\gamma\alpha} + C_\beta\breve{P}^\alpha, \\
\widetilde{L}^\alpha{}_{\dot\beta}(\tau,\vec{\sigma}) &= 2x_{\gamma\dot\beta}\pi^{\alpha\gamma} + \bar{z}_{\dot\beta\dot\gamma}P^{\dot\gamma\alpha} + \bar\theta_{\dot\beta}\pi^\alpha - \tilde{u}^\alpha\tilde{\bar{\breve{P}}}_{u\dot\beta} + 2C_{\gamma\dot\beta}\breve{P}^{\gamma\alpha} + \bar{C}_{\dot\beta\dot\gamma}\breve{P}^{\dot\gamma\alpha} - \bar{C}_{\dot\beta}\breve{P}^\alpha.
\end{aligned}
\tag{34}
$$

The adduced expressions should be complemented by their complex conjugate.

Note that supersymmetry and generalized translation generator densities do not contain any ghost contribution. One can check that the P.B.-commutation relations of the $OSp(1|8)$ superalgebra extended by the ghost contributions coincide with the P.B.-commutation relations of the original $OSp(1|8)$ superalgebra [17].

4 Quantization: nilpotent BRST operator and quantum $OSp(1|8)$ algebra

Upon transition to quantum theory all the quantities entering the converted constraints and $OSp(1|8)$ generator densities are treated as operators that implies a choice of certain ordering for products of noncommuting operators. At the same time the canonical Poisson brackets $\{\mathcal{P}^{\mathfrak{M}}(\vec{\sigma}), \mathcal{Q}_{\mathfrak{N}}(\vec{\sigma}')\}_{P.B.} = \delta_{\mathfrak{N}}^{\mathfrak{M}}\delta^p(\vec{\sigma}-\vec{\sigma}')$ used in [17] transform into (anti)commutators

$$
[\hat{\mathcal{P}}^{\mathfrak{M}}(\vec{\sigma}), \hat{\mathcal{Q}}_{\mathfrak{N}}(\vec{\sigma}')\} = -i\delta_{\mathfrak{N}}^{\mathfrak{M}}\delta^p(\vec{\sigma}-\vec{\sigma}').
\tag{35}
$$

It is necessary to provide further nilpotence of the BRST operator, fulfilment of (anti)commutation relations of the $OSp(1|8)$ superalgebra and its generator (anti)commutativity with the BRST operator ensuring the global quantum invariance of the model. In addition, the Hermiticity of the quantum BRST operator and $OSp(1|8)$ generators has to be supported. The Hermiticity requirement may be manifestly satisfied if we start from the above constructed classical representations for the $OSp(1|8)$ generators and BRST charge in which all coordinates are disposed from the left of momenta, i.e. in the form $\mathcal{QP}$, where $\mathcal{Q}$ and $\mathcal{P}$ are the products of the coordinates and momenta contained in Ω and the generators. Then the operator expressions for the latter are presented in the manifestly Hermitian form composed of the operator products $\frac{1}{2}(\hat{\mathcal{Q}}\hat{\mathcal{P}} + (-)^{\epsilon(\mathcal{Q})\epsilon(\mathcal{P})}(\hat{\mathcal{Q}}\hat{\mathcal{P}})^\dagger)$, where $\epsilon(\mathcal{Q})$ and $\epsilon(\mathcal{P})$ are Grassmannian gradings of these coordinate and momentum monomials.

In particular, we obtain the following Hermitian operator representations

$$
\begin{aligned}
\hat{\tilde{\mathcal{S}}}_\gamma(\tau,\vec{\sigma}) =\ & \tfrac{1}{2}(\hat{z}_{\gamma\delta} - 2i\hat{\theta}_\gamma\hat{\theta}_\delta)\hat{Q}^\delta + \tfrac{1}{2}\hat{Q}^\delta(\hat{z}_{\gamma\delta} - 2i\hat{\theta}_\gamma\hat{\theta}_\delta) \\
& + \tfrac{1}{2}(\hat{x}_{\gamma\dot\delta} - 2i\hat{\theta}_\gamma\hat{\bar\theta}_{\dot\delta})\hat{\bar{Q}}^{\dot\delta} + \tfrac{1}{2}\hat{\bar{Q}}^{\dot\delta}(\hat{x}_{\gamma\dot\delta} - 2i\hat{\theta}_\gamma\hat{\bar\theta}_{\dot\delta}) \\
& + 2i\hat{\theta}_\delta(\hat{\tilde{u}}^\delta\hat{\tilde{\bar{P}}}_{u\gamma} + \hat{\tilde{\bar{P}}}_{u\gamma}\hat{\tilde{u}}^\delta) - 4i\hat{\tilde{u}}^\delta\hat{\bar\theta}_{\dot\delta}\hat{\tilde{\bar{P}}}_{u\gamma} + \tfrac{2}{(\hat{\bar\rho}^\tau)^{1/2}}\hat{f}\hat{\tilde{\bar{P}}}_{u\gamma} \\
& + \hat{C}_\gamma{}^\beta\hat{\tilde{\breve{P}}}_\beta + \hat{C}_{\gamma\dot\beta}\hat{\tilde{\bar{\breve{P}}}}^{\dot\beta} + 2i\hat{\theta}_\delta(\hat{C}_\gamma\hat{\breve{P}}^\delta + \hat{\breve{P}}^\delta\hat{C}_\gamma) + 4i\hat{\bar\theta}^{\dot\delta}\hat{C}_\gamma\hat{\bar{\breve{P}}}_{\dot\delta} \\
& - 4i\hat{\theta}_\delta(\hat{C}_\gamma{}^\beta\hat{\tilde{\breve{P}}}_\beta{}^\delta - \hat{\tilde{\breve{P}}}_\beta{}^\delta\hat{C}_\gamma{}^\beta) + 4i\hat{\bar\theta}^{\dot\delta}\hat{C}_\gamma{}^\beta\hat{\tilde{\breve{P}}}_{\beta\dot\delta} + 2i\hat{\theta}_\delta(\hat{C}_{\gamma\dot\beta}\hat{\tilde{\breve{P}}}^{\dot\beta\delta} - \hat{\tilde{\breve{P}}}^{\dot\beta\delta}\hat{C}_{\gamma\dot\beta}) - 8i\hat{\bar\theta}^{\dot\delta}\hat{C}_{\gamma\dot\beta}\hat{\tilde{\bar{\breve{P}}}}^{\dot\beta}{}_{\dot\delta}
\end{aligned}
\tag{36}
$$

for the classical density $\widetilde{S}_\gamma(\tau, \vec{\sigma})$ (27) and

$$\hat{\tilde{\mathcal{L}}}^\alpha{}_\beta(\tau, \vec{\sigma}) = \tfrac{1}{2}(\hat{x}_{\beta\dot{\beta}}\hat{P}^{\dot{\beta}\alpha} + \hat{P}^{\dot{\beta}\alpha}\hat{x}_{\beta\dot{\beta}}) + (\hat{z}_{\beta\gamma}\hat{\pi}^{\gamma\alpha} + \hat{\pi}^{\gamma\alpha}\hat{z}_{\beta\gamma})$$

$$+ \tfrac{1}{2}(\hat{\theta}_\beta\hat{\pi}^\alpha - \hat{\pi}^\alpha\hat{\theta}_\beta) + \tfrac{1}{2}(\hat{\tilde{u}}^\alpha\hat{\tilde{P}}_{u\beta} + \hat{\tilde{P}}_{u\beta}\hat{\tilde{u}}^\alpha) \tag{37}$$

$$- (\hat{C}_\beta{}^\gamma\hat{\mathring{P}}_\gamma{}^\alpha - \hat{\mathring{P}}_\gamma{}^\alpha\hat{C}_\beta{}^\gamma) + \tfrac{1}{2}(\hat{C}_{\beta\dot{\gamma}}\hat{\mathring{P}}^{\dot{\gamma}\alpha} - \hat{\mathring{P}}^{\dot{\gamma}\alpha}\hat{C}_{\beta\dot{\gamma}}) + \tfrac{1}{2}(\hat{C}_\beta\hat{\mathring{P}}^\alpha + \hat{\mathring{P}}^\alpha\hat{C}_\beta),$$

for the generalized Lorentz density $\widetilde{L}^\alpha{}_\beta(\tau, \vec{\sigma})$ (34).

By the same way can be constructed quantum Hermitian generators $\hat{\tilde{L}}^{ext}_M$ of the $\vec{\sigma}$-reparametrizations and the Weyl symmetry generator $\hat{\tilde{\Delta}}^{ext}_W$

$$\hat{\tilde{\Delta}}^{ext}_W = \tfrac{1}{2}(\hat{\tilde{u}}_\alpha\hat{\tilde{P}}^\alpha_u + \hat{\tilde{P}}^\alpha_u\hat{\tilde{u}}_\alpha + \hat{\tilde{u}}_{\dot{\alpha}}\hat{\tilde{P}}^{\dot{\alpha}}_u + \hat{\tilde{P}}^{\dot{\alpha}}_u\hat{\tilde{u}}_{\dot{\alpha}})$$

$$- \hat{\tilde{\rho}}^\tau\hat{\tilde{P}}^{(\rho)}_\tau - \hat{\tilde{P}}^{(\rho)}_\tau\hat{\tilde{\rho}}^\tau - \hat{\rho}^M\hat{P}^{(\rho)}_M - \hat{P}^{(\rho)}_M\hat{\rho}^M - \hat{C}^{(\rho)M}\hat{\mathring{P}}^{(\rho)}_M + \hat{\mathring{P}}^{(\rho)}_M\hat{C}^{(\rho)M} \approx 0, \tag{38}$$

entering the BRST operator.

Because other converted first-class constraints are Hermitian by construction, the quantum Hermitian BRST generator

$$\hat{\mathbf{\Omega}} = \frac{1}{2}\int d^p\sigma(\hat{\Omega}(\tau, \vec{\sigma}) + \hat{\Omega}^\dagger(\tau, \vec{\sigma})) \tag{39}$$

will coincide with its classical expression Ω (22) after the substitution of (38) and the Hermitian representation for $\hat{\tilde{L}}_M(\vec{\sigma})$ originated from (24) in eq. (22).

Now we are ready to prove that this realization of $\hat{\mathbf{\Omega}}$ preserves its nilpotency and (anti)commutativity with the Hermitian operators (36), (37) and other ones generating a quantum realization of the classical $OSp(1|8)$ superalgebra. The proof is obvious and is based on the observation that $\hat{\mathbf{\Omega}}$ and other considered Hermitian operators are linear in the momentum operators $\hat{\mathcal{P}}^{\mathfrak{M}}(\tau, \vec{\sigma})$ of the original coordinates and ghost fields. The remarkable property of the ordered polynomial operators composed of $\hat{\mathcal{Q}}_{\mathfrak{M}}(\tau, \vec{\sigma})$ and $\hat{\mathcal{P}}^{\mathfrak{M}}(\tau, \vec{\sigma})$, which form the Weyl-Heisenberg algebra (35), and are linear in $\hat{\mathcal{P}}^{\mathfrak{M}}$ is the preservation of the chosen ordering in course of calculations of their (anti)commutators. As a result, the transition from the P.B.'s to (anti)commutators will preserve all classical results obtained in the P.B. realization of the extended algebra of the $OSp(1|8)$ generators and classical BRST charge of the super p-brane. So, the quantum Hermitian BRST operator (39) occurs to be nilpotent

$$\{\hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}\} = 0. \tag{40}$$

However, the Hermiticity of $\hat{\mathbf{\Omega}}$ and the $OSp(1|8)$ generating operators by itself is only a necessary condition for the quantum realization of the physical operators, because the relevant vacuum and physical states have also to be constructed. So, the problem of existence of the selfconsistent quantum realization of the exotic BPS states by the states of quantum tensionless super p-brane is reduced to the proof of existence of the relevant vacuum and the corresponding physical space of quantum states. At the present time we investigate this problem. However, we should like to discuss here a possible way to solve this problem based on the consideration of the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering studied in [10].

5 $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering and regularization

It was motivated in [10] that the coordinate and momenta operators have to be used for the tensionless string quantization instead of the creation and annihilation operators relevant for the tensile string quantization. This motivation is physically justified by the absence of oscillator excitations for the tensionless string which makes its dynamics resembling that of collection of free particles and results in the choice of the physical vacuum as a state annihilated by the string momentum operator. In that case the coordinate $\hat{\mathcal{Q}}$ and momentum $\hat{\mathcal{P}}$ monomials forming the above discussed Hermitian operators have to be ordered by the shifts of all the $\hat{\mathcal{Q}}$ monomials to the left of $\hat{\mathcal{P}}$. To achieve that $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering we have, in particular, to permutate some noncommuting coordinate and momentum operators in the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-disordered Hermitian expressions of the generator densities (36), (37) and others, constraints $\hat{\tilde{\Delta}}_W^{ext}$ (38), $\hat{\tilde{L}}_M^{ext}$ and the Hermitian BRST operator. In view of that permutations divergent terms will appear in some monomials composed from canonically conjugate operators at coinciding points of p-brane. A typical form of such divergent terms is illustrated by the relation

$$\frac{1}{2}(\hat{z}_{\lambda\delta}(\vec{\sigma})\hat{\pi}^{\delta\varepsilon}(\vec{\sigma}) + \hat{\pi}^{\delta\varepsilon}(\vec{\sigma})\hat{z}_{\lambda\delta}(\vec{\sigma})) = \hat{z}_{\lambda\delta}(\vec{\sigma})\hat{\pi}^{\delta\varepsilon}(\vec{\sigma}) - \frac{3i}{4}\delta_\lambda^\varepsilon \delta^p(0) \tag{41}$$

encoding the permutation effect of the TCC coordinates with their momenta. The r.h.s. of ambiguously defined relation (41) includes the divergent term $\delta^p(0) = \delta^p(\vec{\sigma} - \vec{\sigma}')|_{\vec{\sigma}=\vec{\sigma}'}$ and the problem appears how to deal with the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered representation of the symmetric operator in the l.h.s. of (41). Such type a problem is typical in quantum field theory due to its inherit divergencies and the regularization procedure should be applied. Thus, in general, the ordering problem has to be analyzed together with the divergency problem. It might have happened that a regularization prescribes to use the image of the delta function at the zero point (here $\delta^p(0)$) to be equal to zero. Then the choice of that regularization would solve the ordering problem. Taking into account such a possibility requires study of the structure of the divergent terms appearing in the total quantum algebra of our model. To this end we firstly analyze the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered realization of the $OSp(1|8)$ algebra.

We find that the application of the described $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering procedure to the Hermitian $OSp(1|8)$ generator densities (36), (37) and others yields the relations

$$\hat{\tilde{\mathcal{S}}}_\gamma(\tau,\vec{\sigma}) = \hat{\tilde{S}}_\gamma - 2\hat{\theta}_\gamma \delta^p(0), \quad \hat{\tilde{\mathcal{S}}}_{\dot\gamma}(\tau,\vec{\sigma}) = \hat{\tilde{S}}_{\dot\gamma} - 2\hat{\bar\theta}_{\dot\gamma}\delta^p(0),$$

$$\hat{\tilde{\mathcal{K}}}_{\gamma\lambda}(\tau,\vec{\sigma}) = \hat{\tilde{K}}_{\gamma\lambda} + i\hat{z}_{\gamma\lambda}\delta^p(0), \quad \hat{\tilde{\mathcal{K}}}_{\dot\gamma\dot\lambda}(\tau,\vec{\sigma}) = \hat{\tilde{K}}_{\dot\gamma\dot\lambda} + i\hat{\bar{z}}_{\dot\gamma\dot\lambda}\delta^p(0),$$

$$\hat{\tilde{\mathcal{K}}}_{\gamma\dot\gamma}(\tau,\vec{\sigma}) = \hat{\tilde{K}}_{\gamma\dot\gamma} + i\hat{x}_{\gamma\dot\gamma}\delta^p(0), \tag{42}$$

$$\hat{\tilde{\mathcal{L}}}_{\alpha\beta}(\tau,\vec{\sigma}) = \hat{\tilde{L}}_{\alpha\beta} + \tfrac{i}{2}\varepsilon_{\alpha\beta}\delta^p(0), \quad \hat{\tilde{\mathcal{L}}}_{\dot\alpha\dot\beta}(\tau,\vec{\sigma}) = \hat{\tilde{L}}_{\dot\alpha\dot\beta} + \tfrac{i}{2}\bar\varepsilon_{\dot\alpha\dot\beta}\delta^p(0)$$

connecting the Hermitian and $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered representations for the generator densities. The operators $\hat{\tilde{S}}_\gamma$, $\hat{\tilde{S}}_{\dot\gamma}$, $\hat{\tilde{K}}_{\gamma\lambda}$, $\hat{\tilde{K}}_{\dot\gamma\dot\lambda}$, $\hat{\tilde{K}}_{\gamma\dot\gamma}$, $\hat{\tilde{L}}_{\alpha\beta}$, $\hat{\tilde{L}}_{\dot\alpha\dot\beta}$, collectively denoted by $\hat{\tilde{G}}_{qp}$, in the r.h.s. of (42) coincide with the classical $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered representations (27), (28), (32)-(34), where the corresponding operators are substituted for the classical coordinates and momenta. These $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered operators $\hat{\tilde{G}}_{qp}$ form another representation of the quantum algebra $OSp(1|8)$ similarly to the Hermitian operators, because they originate from the classical generators

and preserve the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering in the course of the calculation of their (anti)commutators. But, the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered generators $\hat{\tilde{G}}_{qp}$ are nonHermitian and their (anti)commutation with the divergent nonHermitian terms in the r.h.s. of the representation (42) contributes to the closure of the $OSp(1|8)$ algebra presented by the Hermitian generators in the l.h.s. of (42). So, we obtain two quantum realizations of the $OSp(1|8)$ algebra and one of them $\hat{\tilde{G}}_{qp}$ is nonHermitian, because of its $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering. But, namely, action of the nonHermitian generators $\hat{\tilde{G}}_{qp}$ on the corresponding vacuum state is well defined in accordance with [10]. Then the regularization assumption, which prescribes to accept the regularized image of $\delta^p(0)$ as equal zero, removes the nonHermiticity of the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered generators $\hat{\tilde{G}}_{qp}$. In the result we obtain the desired vacuum state for the discussed Hermitian realization of the physical operators. So, the choice of such a regularization would allow to overcome the problem of construction of the quantum space of physical states. To realize such a scenario one needs to analyze the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered realization of Hermitian BRST generator (39).

The correspondent $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered BRST operator $\hat{\Omega}$ turns out to be nonHermitian, as follows from the relation connecting $\hat{\Omega}$ with the Hermitian BRST operator $\hat{\mathbf{\Omega}}$ (39)

$$\hat{\mathbf{\Omega}} = \hat{\Omega} - i \int d^p\sigma [\hat{C}^{(W)} + (\tfrac{p}{2} - \tfrac{7}{4})\hat{C}^M \partial_M + \tfrac{1}{2}\partial_M \hat{C}^M]\delta^p(0), \qquad (43)$$

and is supplemented by three antiHermitian divergent additions in the r.h.s. compensating the nonHermiticity of $\hat{\Omega}$. The first of them, proportional to $\delta^p(0)$, follows from the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering of the Weyl symmetry generator $\hat{\tilde{\Delta}}_W^{ext}$ (38) and is contributed only by the auxiliary variables $\hat{\tilde{\rho}}^\tau, \hat{\tilde{u}}^\alpha$ and $\hat{\tilde{\bar{u}}}^{\dot\alpha}$ partially cancelling each other during the ordering with their momenta. The contribution of other auxiliary pair $(\hat{\rho}^M, \hat{P}_M^{(\rho)})$ in here is cancelled by the ghost pair $(\hat{C}^{(\rho)M}, \hat{\tilde{P}}_M^{(\rho)})$ contribution. We observe that only the auxiliary twistor-like fields and the component $\hat{\tilde{\rho}}^\tau$ of the world-volume density $\hat{\rho}^\mu$, introduced to provide the Weyl and reparametrization gauge symmetries of the brane action, contributed to the first singular term. The similar story concerns the second divergent term proportional to $\partial_M \delta^p(0)$ [3]. This term appears from the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering of the extended Virasoro operators $\hat{\tilde{L}}_M^{ext}$ and only the above mentioned auxiliary fields together with the ghost pairs $(\hat{C}^M, \hat{\tilde{P}}_M)$, $(\hat{C}^{(W)}, \hat{\tilde{P}}^{(W)})$ and the auxiliary fermionic field $\hat{f}$ contribute to here, because the contributions of the propagating phase-space variables are cancelled by the corresponding ghosts. This cancellation illustrates the boson-fermion cancellation mechanism provided by the BRST symmetry. The third singular addition, proportional to the total derivative, restores Hermiticity of the cubic term $\hat{C}^M \partial_M \hat{C}^N \hat{\tilde{P}}_N$ but, it vanishes in view of the periodical boundary conditions. The latter could contribute in the case on a nontrivial topology of the ghost-field space.

Now let us note that the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered operator $\hat{\Omega}$ in the r.h.s. of eq.(43) is nilpotent

$$\{\hat{\Omega}, \hat{\Omega}\} = 0, \qquad (44)$$

---

[3]In the symmetric regularization, where $\delta_\epsilon^p(-\vec{\sigma}) = \delta_\epsilon^p(\vec{\sigma})$, the derivative $\partial_M \delta_\epsilon^p(\vec{\sigma})$ vanishes at $\vec{\sigma} = 0$. As a result the symmetric regularization does not capture the divergencies following from the reordering of terms with derivatives like $\hat{\pi}^{\delta\varepsilon}(\vec{\sigma})\partial_M \hat{z}_{\lambda\delta}(\vec{\sigma})$ (cf. (41)). To capture such a type singularity one can use more general regularization of delta function considered by Hörmander [25]. As a result, we find the r.h.s. in the regularized (anti)commutators $[\hat{\mathcal{P}}^{\mathfrak{M}}(\vec{\sigma}), \partial_M \hat{\mathcal{Q}}_{\mathfrak{N}}(\vec{\sigma})\} = i\delta_{\mathfrak{N}}^{\mathfrak{M}}\partial_M \delta^p(\vec{\sigma}, \vec{\sigma})$, where $\partial_M \delta^p(\vec{\sigma}, \vec{\sigma})$ is a regularized analogue of $\partial_M \delta^p(0)$, to be non zero.

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because of the nilpotence of the classical BRST charge Ω and its linearity in momenta of $\hat{\Omega}$. Thus, the singular representation (43) contains two nilpotent operators $\hat{\Omega}$ and $\hat{\Omega}$ in view of (40) and (44). Then, the calculation of the anticommutators of the l.h.s. of eq.(43) with itself and the r.h.s. with itself results in the condition

$$\{\hat{\Omega}, \hat{\Omega}\} = 0 = -\int d^p\sigma [2\hat{C}^M \partial_M \hat{C}^{(W)} + (p - \tfrac{7}{2})\hat{C}^M \partial_M \hat{C}^N \partial_N]\delta^p(0). \tag{45}$$

To clarify the expression (45) consider, for simplicity, the contribution of the canonical pair $(\hat{\rho}^\tau, \hat{P}^{(\rho)}_\tau)$ to the anticommutator (45) of the BRST operator (43) using its different representations by the left and right hand sides of (43). For our purpose it is sufficient to consider the contribution to the square of the BRST operator proportional to the product of ghost operators $\hat{C}^{(W)}$ and $\hat{C}^M$. Then for the BRST operator $\hat{\Omega}$ (39) in the l.h.s. of (43) we obtain

$$\tfrac{1}{4}\{\hat{C}^{(W)}(\hat{\rho}^\tau \hat{P}^{(\rho)}_\tau + \hat{P}^{(\rho)}_\tau \hat{\rho}^\tau)(\vec{\sigma}), -\hat{C}^M(\hat{\rho}^\tau \partial_{M'} \hat{P}^{(\rho)}_\tau + \partial_{M'}\hat{P}^{(\rho)}_\tau \hat{\rho}^\tau)(\vec{\sigma}') +$$
$$\hat{C}^M(\partial_{M'}\hat{C}^{(W)} \hat{\tilde{P}}^{(W)} - \hat{\tilde{P}}^{(W)} \partial_{M'}\hat{C}^{(W)})(\vec{\sigma}')\}. \tag{46}$$

The desired contributions come from the two anticommutators in (46)

$$\tfrac{1}{4}\{\hat{C}^{(W)}(\hat{\rho}^\tau \hat{P}^{(\rho)}_\tau + \hat{P}^{(\rho)}_\tau \hat{\rho}^\tau)(\vec{\sigma}), -\hat{C}^M(\hat{\rho}^\tau \partial_{M'} \hat{P}^{(\rho)}_\tau + \partial_{M'}\hat{P}^{(\rho)}_\tau \hat{\rho}^\tau)(\vec{\sigma}')\}$$
$$= -\tfrac{i}{2}\hat{C}^{(W)}(\vec{\sigma})\hat{C}^M(\vec{\sigma}')(\hat{\rho}^\tau(\vec{\sigma}')\hat{P}^{(\rho)}_\tau(\vec{\sigma}) + \hat{P}^{(\rho)}_\tau(\vec{\sigma})\hat{\rho}^\tau(\vec{\sigma}'))\partial_{M'}\delta^p(\vec{\sigma} - \vec{\sigma}') \tag{47}$$
$$+\tfrac{i}{2}\hat{C}^{(W)}(\vec{\sigma})\hat{C}^M(\vec{\sigma}')(\hat{\rho}^\tau(\vec{\sigma})\partial_{M'}\hat{P}^{(\rho)}_\tau(\vec{\sigma}') + \partial_{M'}\hat{P}^{(\rho)}_\tau(\vec{\sigma}')\hat{\rho}^\tau(\vec{\sigma}))\delta^p(\vec{\sigma} - \vec{\sigma}').$$

Transforming the arguments of the multipliers in front of the partial derivative $\partial_{M'}\delta^p(\vec{\sigma} - \vec{\sigma}')$ to be coincident and equal $\vec{\sigma}'$, we find the equivalent representation for the r.h.s of Eq.(47) as follows

$$-\frac{i}{2}\hat{C}^{(W)}\hat{C}^M(\hat{\rho}^\tau \hat{P}^{(\rho)}_\tau + \hat{P}^{(\rho)}_\tau \hat{\rho}^\tau)(\vec{\sigma}')\partial_{M'}\delta^p(\vec{\sigma} - \vec{\sigma}') - \frac{i}{2}\partial_M\hat{C}^{(W)}\hat{C}^M(\hat{\rho}^\tau \hat{P}^{(\rho)}_\tau + \hat{P}^{(\rho)}_\tau \hat{\rho}^\tau)\delta^p(\vec{\sigma} - \vec{\sigma}'). \tag{48}$$

Correspondingly, for the second term we obtain

$$\tfrac{1}{4}\{\hat{C}^{(W)}(\hat{\rho}^\tau \hat{P}^{(\rho)}_\tau + \hat{P}^{(\rho)}_\tau \hat{\rho}^\tau)(\vec{\sigma}), \hat{C}^M(\partial_{M'}\hat{C}^{(W)} \hat{\tilde{P}}^{(W)} - \hat{\tilde{P}}^{(W)}\partial_{M'}\hat{C}^{(W)})(\vec{\sigma}')\}$$
$$= \tfrac{i}{2}\partial_M\hat{C}^{(W)}\hat{C}^M(\hat{\rho}^\tau \hat{P}^{(\rho)}_\tau + \hat{P}^{(\rho)}_\tau \hat{\rho}^\tau)\delta^p(\vec{\sigma} - \vec{\sigma}'). \tag{49}$$

The sum of the contributions (48) and (49) is the total divergence

$$\frac{i}{2}\partial_M \left(\hat{C}^{(W)}\hat{C}^M(\hat{\rho}^\tau \hat{P}^{(\rho)}_\tau + \hat{P}^{(\rho)}_\tau \hat{\rho}^\tau)\right)\delta^p(\vec{\sigma} - \vec{\sigma}') \tag{50}$$

after the transfer of the derivative $\partial_{M'}$ from the δ-function to the phase-space operators that is a correct operation for distributions.

Thus for the Hermitian representation, the contribution of the canonical pair $(\hat{\rho}^\tau, \hat{P}^{(\rho)}_\tau)$ entering the Weyl generator to the anticommutator $\{\hat{C}^{(W)}\hat{\Delta}^{ext}_W(\vec{\sigma}), \hat{C}^M \hat{L}^{ext}_M(\vec{\sigma}')\}$ vanishes in agreement with the nilpotency of the BRST operator. The same result can be found for the contributions of other canonical pairs.

Now we are interested in the same contribution to the square of the BRST operator proportional to the product of $\hat{C}^{(W)}$ and $\hat{C}^M$ ghosts, but in the QP-ordered and singular realization given by

$$\{\left(\hat{C}^{(W)}\hat{\rho}^\tau\hat{P}_\tau^{(\rho)} - i\hat{C}^{(W)}\delta^p(0)\right)(\vec{\sigma}), -\left(\hat{C}^M\hat{\rho}^\tau\partial_{M'}\hat{P}_\tau^{(\rho)} - \hat{C}^M\partial_{M'}\hat{C}^{(W)}\breve{P}^{(W)}\right)(\vec{\sigma}')\}, \qquad (51)$$

where we omitted the singular terms proportional to $\hat{C}^M\partial_M\delta^p(0)$ in the r.h.s. of the anticommutator (51) which do not contribute into the anticommutator. Then for the r.h.s. of Eq. (43) we obtain the three summands

$$\{(\hat{C}^{(W)}\hat{\rho}^\tau\hat{P}_\tau^{(\rho)})(\vec{\sigma}), -(\hat{C}^M\hat{\rho}^\tau\partial_{M'}\hat{P}_\tau^{(\rho)})(\vec{\sigma}')\}$$
$$= -i(\hat{C}^{(W)}\hat{C}^M\hat{\rho}^\tau\hat{P}_\tau^{(\rho)})(\vec{\sigma}')\partial_{M'}\delta^p(\vec{\sigma} - \vec{\sigma}') - i\partial_M\hat{C}^{(W)}\hat{C}^M\hat{\rho}^\tau\hat{P}_\tau^{(\rho)}\delta^p(\vec{\sigma} - \vec{\sigma}'), \qquad (52)$$

$$\{(\hat{C}^{(W)}\hat{\rho}^\tau\hat{P}_\tau^{(\rho)})(\vec{\sigma}), (\hat{C}^M\partial_{M'}\hat{C}^{(W)}\breve{P}^{(W)})(\vec{\sigma}')\} = i\partial_M\hat{C}^{(W)}\hat{C}^M\hat{\rho}^\tau\hat{P}_\tau^{(\rho)}\delta^p(\vec{\sigma} - \vec{\sigma}').$$

The sum of the above two summands is again the total divergence

$$i\partial_M\left(\hat{C}^{(W)}\hat{C}^M\hat{\rho}^\tau\hat{P}_\tau^{(\rho)}\right)\delta^p(\vec{\sigma} - \vec{\sigma}'), \qquad (53)$$

which can be omitted for the discussed model of the closed p-brane. So, the value of the anticommutator is determined by the singular term $i\hat{C}^{(W)}\delta^p(0)$ contribution equal

$$\{-i\hat{C}^{(W)}(\vec{\sigma})\delta^p(0), (\hat{C}^M\partial_{M'}\hat{C}^{(W)}\breve{P}^{(W)})(\vec{\sigma}')\} = -\delta^p(0)\hat{C}^M\partial_M\hat{C}^{(W)}\delta^p(\vec{\sigma} - \vec{\sigma}'). \qquad (54)$$

One can see that the term does not transform to the total divergence.

Analogous calculations of the contributions of other canonical pairs entering the Weyl generator will also give the same divergent answer modulo numerical coefficients. Summarizing all that contributions we arrive at the first summand in the r.h.s. of (45).

Similarly, calculation of the (anti)commutators of the $OSp(1|8)$ operators (42) with $\hat{\boldsymbol{\Omega}}$ (43) gives the relations

$$\begin{aligned}
\{\hat{\boldsymbol{\Omega}}, \hat{\tilde{\mathcal{S}}}_\gamma\} = 0 =\ & 2i\int d^p\sigma(\hat{C}_\gamma + \hat{C}^M\partial_M\hat{\theta}_\gamma)\delta^p(0), \\
\{\hat{\boldsymbol{\Omega}}, \hat{\tilde{\mathcal{S}}}_{\dot{\gamma}}\} = 0 =\ & -2i\int d^p\sigma(\hat{\tilde{C}}_{\dot{\gamma}} - \hat{C}^M\partial_M\hat{\theta}_{\dot{\gamma}})\delta^p(0), \\
[\hat{\boldsymbol{\Omega}}, \hat{\tilde{\mathcal{K}}}_{\gamma\lambda}] = 0 =\ & -2i\int d^p\sigma[(\hat{C}_\gamma\hat{\theta}_\lambda + \hat{\theta}_\gamma\hat{C}_\lambda) + \tfrac{i}{2}\hat{C}^M\partial_M\hat{z}_{\gamma\lambda}]\delta^p(0), \\
[\hat{\boldsymbol{\Omega}}, \hat{\tilde{\mathcal{K}}}_{\gamma\dot{\gamma}}] = 0 =\ & -2i\int d^p\sigma[(\hat{C}_\gamma\hat{\tilde{\theta}}_{\dot{\gamma}} - \hat{\theta}_\gamma\hat{\tilde{C}}_{\dot{\gamma}}) + \tfrac{i}{2}\hat{C}^M\partial_M\hat{x}_{\gamma\dot{\gamma}}]\delta^p(0).
\end{aligned} \qquad (55)$$

The conditions (45) and (55) contain the divergent terms proportional to $\delta^p(0)$ and $\partial_M\delta^p(0)$, so to attach meaning to them one should assign values to the singular limits of $\delta^p(\vec{\sigma} - \vec{\sigma}')|_{\vec{\sigma}=\vec{\sigma}'}$ and its derivatives, i.e. to make their regularization. In accordance with the regularization the (anti)commutation relations (35) have to be changed by the regularized relations

$$[\hat{\mathcal{P}}^{\mathfrak{M}}(\vec{\sigma}), \hat{\mathcal{Q}}_{\mathfrak{N}}(\vec{\sigma}')\} = -i\delta_{\mathfrak{N}}^{\mathfrak{M}}\delta^p(\vec{\sigma}, \vec{\sigma}'), \qquad (56)$$

where the distribution $\delta^p(\vec{\sigma}, \vec{\sigma}')$ is a regularized image of the Dirac delta function $\delta^p(\vec{\sigma} - \vec{\sigma}')$. The substitution of the distributions $\delta^p(\vec{\sigma}, \vec{\sigma}')$ and $\partial_M\delta^p(\vec{\sigma}, \vec{\sigma}')$ in eqs. (45) and (55) results in their solutions

$$\delta^p(\vec{\sigma}, \vec{\sigma}) = 0, \quad \partial_M\delta^p(\vec{\sigma}, \vec{\sigma}) = 0. \qquad (57)$$

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The regularization that requires $\delta^p(\vec\sigma, \vec\sigma) = 0$ is well known in the physical literature (see, for instance, [26]). This regularization prescription leads to the closure of the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered and regularized superalgebra of local and global symmetries of the considered brane model. The regularization (57) removes the problem of ordering in the operators obtained from the classical expressions and differing from each other by the terms proportional to $\delta^p(0)$ and/or $\partial_M\delta^p(0)$. It shows the possibility of quantization of the super p-brane model in this regularization.

6 Conclusion

The general problem of quantum brane realization of the BPS states preserving $\frac{\text{M}-1}{\text{M}}$ fraction of partially spontaneously broken global $N = 1$ supersymmetry at the classical level was analyzed on example of the twistor-like p-brane in four-dimensional space-time. Twistor-like brane models are characterized by a fine tuning of a large set of classical local and global symmetries caused by the absence of tension. We constructed classical BRST charge and generators of these global and gauge symmetries and proved the closure of their unified P.B. superalgebra. The P.B. realization of the nilpotency condition for the BRST charge and its (anti)commutativity with the unified symmetry generators were proved. After that we considered quantization of the model and proved the preservation of the above classical results using Hermitian operator realization of the symmetry generators and BRST operator. This Hermitian realization gives the relevant physical foundation for the solution of the quantization problem. The remaining problem here is the construction of the vacuum and the Hilbert space of quantum states of tensionless super p-branes.

Otherwise, such a type problem was earlier considered in [10] using the general construction of the finite inner product [11] and the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering in the quantum BRST charge and generators of the symmetry algebra of bosonic tensionless string. There was shown the existence of the full physical vacuum that is annihilated by the BRST and string momentum operators. The latter condition has picked up the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered representation as a physical one for tensionless objects. Motivated by these results we studied the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered operator realization of the brane symmetry generators and the BRST operator. It was found the closure of the quantum generalized superconformal algebra $OSp(1|8)$ in this realization and the (anti)commutativity of the quantum $OSp(1|8)$ generators with nilpotent BRST operator, because of the linearity of the quantum operators in the momentum operators $\hat{\mathcal{P}}^{\mathfrak{M}}$. But, the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered representation for the $OSp(1|8)$ generators $\hat{\tilde{G}}_{qp}$ and the BRST operator $\hat\Omega$ turn out to be nonHermitian. We remind that the latter problem might be solved by the definite choice of the regularization assumption for the delta function $\delta^p(0)$ and its derivatives $\partial_M\delta^p(0)$. It shows a possibility to quantize the classical super p-brane preserving $\frac{3}{4}$ fraction of partially spontaneously broken $D = 4$ $N = 1$ supersymmetry in the special regularization.

Our consideration admits straightforward generalization to the case of the space-time dimensions D belonging to the set $2, 3, 4(mod\,8)$ consistent with the Majorana spinor existence. Taking into account that the number coefficients in front of the divergent terms containing $\delta^p(0)$ and $\partial_M\delta^p(0)$ depend on D one can hope that they might be equal to zero for some D from the above mentioned set. It would prove the existence of the quantized model outside the regularization prescription. The above mentioned problems are under our investigation.

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7 Acknowledgements

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